DESCRIPTIVE GEOMETRY IN TODAY'S ENGINEERING CURRICULUM

Summary

The paper deals with Descriptive Geometry, a subject of basic importance for any engineering education. Descriptive Geometry has always been a method to study 3D geometry through 2D images, thus offering an insight into the structure and metrical properties of spatial objects, processes and principles. The education in Descriptive Geometry provides a training of the student’s intellectual capability of space perception. Drawings are a guide to geometry but not the main aim.

Key words: Descriptive Geometry, space perception

1. Introduction

The aim of this paper is to explain what Descriptive Geometry is good for. This is a subject which in the hierarchy of sciences is placed somewhere within or next to the field of Mathematics, but also close to Architecture, Mechanical Engineering, and Engineering Graphics. I start with definitions and continue with a few examples in order to highlight the fact that Descriptive Geometry provides a training of the student’s intellectual capability of space perception (note the diagram in Fig. 9) and is therefore of incontestable importance for all engineers, physicians and natural scientists.

2. How to define ‘Descriptive Geometry’

In many American textbooks on Engineering Graphics, e.g. [2, part III] or [6], the subject of Descriptive Geometry seems to be restricted to standard constructions, such as the determination of the true length of a line segment or the intersection of two plane polygons in 3-space. From this point of view, it must look rather strange that prominent geometers have devoted their whole academic life to promote this subject.

2.1 Descriptive Geometry in Europe

In order to explain the meaning of ‘Descriptive Geometry’ in central Europe, let us look for definitions in German textbooks published in the last five decades:

• J. Krames defined in [9]: “Descriptive Geometry is the high art of spatial reasoning and its graphic representation”.

This definition has also been cited by R. Bereis in [1].
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• H. Brauner took up a recommendation given by E. Kruppa and preferred the designation ‘Constructional Geometry’ [Konstruktive Geometrie] instead of Descriptive Geometry. He defined in [4]: “Constructional Geometry encompasses the analysis of 3D objects by means of graphical or mathematical methods applied to 2D images.”

• F. Hohenberg, whose textbook [7] focuses on the applications of Descriptive Geometry in technology, formulated: “Constructional Geometry teaches how to grasp, to imagine, to design, and to draw geometrical shapes.”

• W.-D. Klix gives the following extended explanation in his recent textbook [8]: “Descriptive Geometry is unique in the way how it promotes spatial reasoning, which is so fundamental for each creative activity of engineers, and how it trains the ability to express spatial ideas graphically so that they become understandable for anybody else.”

As a consequence, I would like to summarize these definitions in the following way:

**Definition:** ‘Descriptive Geometry’ is a method to study 3D geometry through 2D images. It provides an insight into the structure and metrical properties of spatial objects, processes and principles. Typical for Descriptive Geometry is the interplay

a) between a 3D situation and its 2D representation, and

b) between intuitive grasping and rigorous logical reasoning.

According to this definition, courses of Descriptive Geometry in central Europe cover not only the projection theory, but also modelling techniques for curves, surfaces, and solids, thus offering an insight into a broad variety of geometric shapes. Besides, an intuitive approach to elementary differential-geometric properties of curves and surfaces and some 3D analytic geometry is included ([12, 13]). In addition, one of its aims is also to develop and to refine the student’s problem-solving skills.

2.2 G. Monge’s definition

Gaspard Monge (1746–1818) is considered to be the founder of the science of Descriptive Geometry. This does not mean that he himself developed all the graphical methods. On the contrary, most of them can be found in earlier books, e.g., in those of Amédée François Frezier. However, G. Monge was a most effective scientist and manager who spreaded his ideas of Descriptive Geometry with the publication of his ‘Leçons de géométrie descriptive’ (1799) from France over the whole Europe.

We find in [10], p. 1, the following introductory statements:

“La Géométrie descriptive a deux objets:

• le premier, de donner les méthodes pour représenter sur une feuille de dessin qui n’a que deux dimensions, savoir, longueur et largeur, tous les corps de la nature qui en ont trois, longueur, largeur et profondeur, pourvu néanmoins que ces corps puissent être définis rigoureusement.

**Fig. 1** Statue of G. Monge
Place de Monge, Beaune (birthplace)
Dep. Côte-d’Or, France
This proves that the two main objectives of Descriptive Geometry – imaging and analysing 3D objects – date back to the founder. These two targets can also be found in new encyclopaedias, such as Brockhaus [5]:

“Descriptive Geometry = subject of mathematics, … The aim of DG is the representation of 3D objects … as well as the interpretation of given images … “

This fixture is made from mild steel and consists of a rectangular block 75mm high, 44mm long and 100mm wide. It has a 25mm thick by 100mm wide flange protruding from the 100mm face of the block, with the lower surfaces (base) aligned. The free end of the flange is rounded with a 50mm radius and at the centre of that radius is a hole through the flange, 8mm in diameter, with a counterbore, 20mm in diameter, 10mm deep in the top surface of the flange. The overall length of the fixture is 150mm.

The rectangular block has a V-slot shaped symmetrically through the top surface in a longitudinal direction. It is 38mm each side of the centre at the top surface and is 45 degrees to this surface. The bottom of the V-slot is removed by a rectangular slot 19mm wide, with its bottom face 10mm above the top face of the flange.

Fig. 2 On the importance of graphic representations – one illustration versus ‘1000 words’.
Source: K. Suzuki [15, Fig. 1] (with permission of Heldermann Verlag)

2.3 The choice of the name

It is remarkable that the word ‘drawing’ does not appear in Monge’s definition. In Descriptive Geometry drawing¹ is a guide to geometry (compare [14]) but not the main aim; we teach geometry instead of construction techniques. Note that the French ‘descriptive’ means ‘describing’, ‘representing’ but not necessarily ‘graphically depicting’.

Nevertheless, in the public opinion Descriptive Geometry has falsely become synonymous with ‘manually drawn images of 3D objects’. As in the last decades manual drawing using traditional instruments has been replaced by CAD or mathematical software with graphic output, ‘people on the road’ frequently conclude that consequently Descriptive Geometry has become obsolete.

However, this is totally wrong. On the contrary, because

• only the people with a profound knowledge in Descriptive Geometry are able to make an extended use of CAD programs as the communication is usually based on views only.

• the more powerful and sophisticated modelling software is, the higher is the required geometric knowledge.

• a poor designer will never become perfect only by using CAD instead of traditional tools.

¹ It is said that Felix Klein once stated: “Among all mathematicians, geometers have the advantage to see what they are studying.”
For similar reasons, the importance of mathematics is still increasing though computers take over the computational labour.

Another misinterpretation of Descriptive Geometry is to consider it only as a theoretical, rather ‘academic’ subject. F. Hohenberg could disprove this opinion in his textbook [7] in a convincing way. In many examples he demonstrated the application of Descriptive Geometry to real-world requirements.

In order to defend the true meaning of Descriptive Geometry, there have been various attempts to rename this subject. Its applicability is stressed by using the names ‘Technical Geometry’ or ‘Applied Geometry’ instead of ‘Descriptive Geometry’. As already mentioned, another choice is ‘Constructive Geometry’ – ‘constructive’ in its figurative sense. It should indicate that not only manual drawings but also mathematical computations are used in this subject.

Anyway, the original Monge’s definition of ‘Descriptive Geometry’ with its wide meaning covers all these aspects. So, in my opinion, the original name is still appropriate. However, some find this name old-fashioned. For strategic reasons they are seeking for more attractive designations which make evident that temporary courses on Descriptive Geometry include also some methods from computer science, such as ‘geometric modelling’ as well as ‘visualization techniques’ and, of course, CAD-programs. In this sense ‘Geometric Modelling and Visualization’ or more briefly ‘Modelling and Imaging’ could be appropriate and more in fashion.

And for those who like to translate ‘descriptive’ by ‘graphically depicting’ only, I add the following statement: ‘Descriptive Geometry’ is more than ‘descriptive’ geometry as well as ‘Geometry’ is more than its literal sense, i.e., ‘measuring the earth’.

3. What should remain in the student’s brain

In order to estimate the educational effect of any subject included in a curriculum, one should try to figure out what remains in the student’s brain after all details have already been forgotten. I would like to state that even for poor students, the education in Descriptive Geometry brings about the ability

- to comprehend spatial objects from given principal views, and
- to specify and grasp particular views. Besides,
- the students get an idea of geometric idealization (abstraction), of the variety of geometric shapes, and of geometric reasoning.

The first two items seem to be rather elementary. However, these intellectual abilities are so fundamental that many people forget later how hard it is to achieve these abilities.

3.1 The importance of principal views

Familiarity with the principal views – top view, front view, and side view – is substantial for several reasons:

- They are more or less abstract as they do not correspond to our personal visual impression. But abstraction simplifies.
- In the majority of cases they make the essential properties of spatial structures obvious, and
- It is much easier to inspect these planar views than to concentrate on the original spatial structure.

Fig. 3 Explanation of principal views in a textbook for dentists
There is no better way to explain the baby’s 3D movement when being born. However, it needs training to become familiar with this kind of representation and to grasp the shape of any 3D object just by looking at its principal views. Nobody questions the necessity of a permanent training for sportsmen. But in case of Descriptive Geometry, people often neglect this necessity and they speak of a purely academic subject, when, e.g., in introductory exercises two triangles in space are to intersect.

Medical doctors often hold in esteem their Descriptive Geometry education. In anatomy, they could much easier comprehend the course of blood-vessels or nerves just by sketching them in the principal views. And in orthopedics, they were better able to grasp how human joints are operating and why dislocations have specific consequences.

A few months ago Austrian television was broadcasting a life operation on a human skull. The surgeon had to correct a dislocation of the cheek-bone, caused by a traffic accident. In a pre-operative step the required position of the cheek-bone was already marked on a screen. By an image-fusion this virtual target posture was combined with the actual one. So, the surgeon's work consisted of making these two positions coincident by manual manipulations on the patient.

How did the surgeon control his work? He inspected the three principal views as they allowed decomposing the true 3D displacement into (more or less) planar motions.

### 3.2 The art of specifying particular views

**Axonometric views** are important and well understandable for everybody. And they are appropriate to remember on a known object or to compare with a real object nearby. However, no angle, no length, no planar shape appears in its real size. Orthogonality can be figured out only because of some additional assumptions based on experience or estimation. So, these views are never sufficient for a ‘description exacte’ as required in Monge’s definition.3

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2 The author is grateful to Prof. A. Schmid-Kirsch, University Hannover, for submitting Fig. 4.

3 The same is true for illustrations shaded like photographs. They can be very impressive, but also extremely cheating. Pure line graphics look less attractive; they offer a sort of data compression and they are more abstract. But often this is an advantage as much more information is included, and line graphics allow concentrating on what is essential.
For a detailed analysis of a 3D object, *particular views* (auxiliary views) with planes in edge view or lines in point view can really reveal the spatial situation. Such views are often the key to the solution of a 3D problem. In my opinion, these particular views make the *sublime art* of Descriptive Geometry. Only in Descriptive Geometry courses students learn what conditions can be simultaneously fulfilled in particular views and how such views can be specified.

The following example (Fig. 5, cf. [11]) will demonstrate the advantage of particular views:

**Example:** *Where does the sun rise earlier on June 21, in Oslo or in Vienna?*

<table>
<thead>
<tr>
<th>city</th>
<th>Eastern longitude</th>
<th>Northern latitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oslo</td>
<td>10.6°</td>
<td>59.9°</td>
</tr>
<tr>
<td>Vienna</td>
<td>16.4°</td>
<td>48.2°</td>
</tr>
</tbody>
</table>

We specify a front view in Fig. 5 with sun rays parallel to the image plane. Then we assume that this view is taken at the moment when the sun is rising in Oslo on June 21. As soon as Vienna is displayed in this view, we see at the first glance the answer to the posed question.

The same view is also useful for clearing additional and more detailed problems such as the following:

a) Can it happen over the period of one year that the sun rises simultaneously in Oslo and Vienna?

b) We increase the precision by paying attention to the fact that because of refraction in the atmosphere, for the observer on earth, the sun seems to rise while it is still approx. 0.6° under the local horizon.

c) In the *zone of astronomic dawn* the sun is between 6° and 18° under the local horizon. Inspecting the particular view presented above, it is easy to comprehend why the period of the daily dawn is shorter when the observer is nearer to the equator.

### 3.3 Views are a guide to spatial geometry

I do not know if anybody is able to manipulate virtual 3D objects – without any tools, in his imagination only – and to figure out how these objects look like in different postures. Maybe, sculptors or pilots have this mental ability. Actually, I myself do not – despite highest mental exertion. And the rhombic dodecahedron serves as a convincing example for me.

This convex polyhedron can be built by erecting quadratic pyramids with 45° inclined planes over each face of a cube (Fig. 6). As any two coplanar triangles can be glued together
forming a rhomb, this polyhedron has 12 congruent faces and seems to be well understood. Nevertheless, I am not able to imagine (with my eyes closed) how this polyhedron looks like from above when it is resting with one face on a table. Fortunately, a simple freehand sketch helps to figure out this view as well as other remarkable properties such as the following:

- There are two types of vertices at the rhombic dodecahedron: 8 vertices belong to the initial cube; the other 6 are mirror images of the cube's center under reflection in the faces.
- The rhombic dodecahedron is the intersection of 3 quadratic prisms with pair-wise orthogonal axes (see Fig. 7).

![Fig. 6 Cube and rhombic dodecahedron](image)

![Fig. 7 Rhombic dodecahedron as an intersection of three quadratic prisms](image)

![Fig. 8 Different views of a rhombic dodecahedron](image)

- The rhombic dodecahedron is an intersection of hexagonal prisms with axes placed on cube-diagonals. There are chains of 6 adjacent faces (note the shaded rhombs in Fig. 8) which are located on the same hexagonal prism.
- The side and back walls of a honey comb belong to a rhombic dodecahedron.
- Each dihedral angle makes 120°, and there is an in-sphere (contacting all edges of the initial cube).
- The rhombic dodecahedron\(^4\) is dual to the cuboctahedron.
- The rhombic dodecahedron is a space-filling polyhedron. This can be figured out by starting with a ‘3D-chessboard’ built from cubes. Then the ‘white’ cubes can be partitioned into 6 quadratic pyramids. Each can be added to the adjacent ‘black’ cube, thus enlarging it to a rhombic dodecahedron.

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\(^4\) More strictly, it should be called first rhombic dodecahedron. Due to S. Bilinski [3] there is a second one: in this case the dihedral angle is 144°. This polyhedron is obtained from the triacontahedron, the dual of the icosidodecahedron, by removing two prismatic zones and bringing the remaining pieces together. The author is grateful to H. Martini for drawing his attention to this fact.
4. Descriptive Geometry in presence of computers

- Descriptive Geometry
  - Computer Graphics,
  - Engineering Graphics,
  - 3D-CAD,
  - control persons

Source: K. Suzuki [15, Fig. 5] (with permission of Heldermann Verlag)

![Fig. 9 Differences between pre- and post-MCT-test on Japanese students before and after the graphics education](image)

The statistics given in Fig. 9 reveals the high effect of traditional Descriptive Geometry education on improving spatial ability. However, one should permanently try to present topics in an up-dated form, so that they are attractive for the majority of young people.

The following tables summarize some of these aspects:

<table>
<thead>
<tr>
<th>What is obsolete:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• complicated manual constructions,</td>
</tr>
<tr>
<td>• hard theoretical proofs,</td>
</tr>
<tr>
<td>• the theory of how to obtain images of particular 3D objects</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>What is still necessary:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• ‘3D-competence’, i.e.,</td>
</tr>
<tr>
<td>• the capability to comprehend virtual 3D situations from given images,</td>
</tr>
<tr>
<td>• mental orientation in 3-space (e.g., the user’s coordinate system),</td>
</tr>
<tr>
<td>• basic knowledge of 3D geometry,</td>
</tr>
<tr>
<td>• promoting creativity and problem-solving skills,</td>
</tr>
<tr>
<td>• applications of geometry,</td>
</tr>
<tr>
<td>• producing attractive illustrations.</td>
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</tbody>
</table>

But one must not forget that there are additional demands on Descriptive Geometry courses:

<table>
<thead>
<tr>
<th>Additional demands:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Handling software for geometric modelling and visualization,</td>
</tr>
<tr>
<td>• treating new geometric shapes (e.g., B-spline surfaces),</td>
</tr>
<tr>
<td>• competence in handling graphics files (in different format),</td>
</tr>
<tr>
<td>• design of animations.</td>
</tr>
</tbody>
</table>
REFERENCES


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